On an EPQ Model with Generalized Pareto Rate of Replenishment and Dterioration with Constant Demand

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Abstract—Inventory models create lot of interest due to their ready applicability in analyzing situations at several places such as market yards, warehouses, raw material scheduling, production control, civil supplies etc. The major string in developing the Inventory models is characterizing the constituent processes such as replenishment (production), demand and life time distribution of the commodity. In this paper we develop and analyze an EPQ model with the assumption that the production quantity is random and follows a Generalized Pareto distribution. It is also further assumed that life time of the commodity is random and follows a Generalized Pareto distribution. The demand rate is constant. Assuming that the shortages are allowed and fully backlogged the instantaneous state of inventory is derived. With suitable cost considerations the total cost function is obtained and minimized for obtaining the production schedules as well as production quantity. Through sensitivity analysis it is observed that the production quantities distribution parameters, the deterioration distribution parameters and demand rate have significant influence on optimal production quantities as well as optimal production up and down times. This model is extended to the case of without shortages. A comparative study of the two models having with and without revealed shortages that allowing shortages will reduce total production cost. This model also includes some of the earlier model as particular cases for specific values of the parameters.

Keywords— EPQ model , Generalized Pareto distribution, Rate of deterioration , constant demand.

1 INTRODUCTION

EPQ models can be applied to a system where a product is product by a production line. The EPQ models are more common in production and manufacturing processes, warehouses, cement industries etc. In EPQ models the major string is relaxing some of the assumptions regarding production (replenishment), lifetime of the commodity and demand pattern (Osteryoung ,et al. [1], Goyal,et al. (2004) [2], Hu and Liu [3] , Srinivasa Rao etal [4])

Recently, much emphasis was given for analyzing EPQ models for deteriorating items. Deterioration is a natural phenomenon of several commodities. The deterioration is highly influenced by several random factors like storage facility, temperature, environmental conditions, quality of raw materials etc. Several authors developed various EPQ models for deteriorating items with various assumptions on lifetime of commodity. Nahmias [5], Raafat [6], Goyal and Giri [7], Ruxian Lie, et al. [8] and Pentico and Drake [9] have reviewed the literature on inventory model for deteriorating items. To develop the EPQ models it is needed to ascribe a probability distribution of the lifetime of commodity. Ghare and Schrader [10], Shah and Jaiswal [11], Cohen [12], Aggarwal [13], Dave and Shah [14], Pal [15], Kalpakam and Sapna [16], Griri and Chaudhuri [17] assumed that the lifetime of commodity follows an exponential distribution. Tadikamalla [18] assumed Gamma distribution to the lifetime of commodity.

Covert and Philip [19], Philip [20], Goel and Aggarwal [21], VenkataSubbaiah,et al. [22], assumed Weibull distribution to the lifetime of commodity . Nirupama Devi [23], et al. developed the inventory models with mixture of Weibull distribution for the lifetime of commodity,Srinivas Rao,et al. [24] developed inventory model with generalized Pareto lifetime, Xu and Li [25] developed a two-warehouse inventory model for deteriorating items with time- dependent demand, Rong,etal.[26], studied a two-warehouse inventory model for deteriorating items with partially / fully backlogged shortages and fuzzy Lead time, Srinivas Rao,et al. [27] studied an inventory model for deteriorating items having additive exponential lifetime and selling price dependent demand rate, Chang and Lin [28] studied an inventory model for deteriorating items with stock dependent demand, Biswajit Sarkar [29] developed and EOQ model with delay in payments and stock dependent demand in presence of imperfect production. However, in all these EOQ models, it is assumed that the replenishment is instantaneous

Mukherjee and Pal [30], Sujit and Goswami [31], Goyal and Giri [32] developed inventory models with finite rate of replenishment (production). Panda and Chatarjee [33], Mandal and Phajudar [34] and Sana,et al. [35] developed inventory models with uniform rate of replenishment (production). Perumal and Arivarignan [36] considered two rates of production in an inventory model. Pal and Mandal [37] and sen and Chakrabarthy [38] developed alternating replenishment rates. Lin and Gong [39], Maity,et al. [40],Hu and Liu [41], Uma MaheshwaraRao,et al. [42] have developed inventory models for deteriorating items with constant rate of production (replenishment). Venkata subbaia, et al. [43] have developed EPQ model with alternating rate of replenishment, Essay and Srinivas Rao [44] have developed EPQ models with stock dependent production and Weibull decay. Ardak and Borade [45] developed an EPQ model for deteriorating items assuming that deterioration of the items start after some constant time as it enters into the inventory.

However, in many production lot size models the production of replenishment rate is not constant or uniform and will have a variable rate of production / replenishment, since the production or replenishment is influenced by several random factors like transportation, quality of raw materials availability, power supply packaging, environmental conditions etc. For example in case of sea food's and agricultural products the uncertainty in the yield effects the procurement as a result production. Also it can be observed several reproduction processes dealing with perishable items will have a random rate of production / replenishment. For modelling this sort of situation it is needed to consider that the replenishment / production is random and follows a probability distribution.

Recently Sridevi, et al. [46], Srinivasarao et al. [47], lakshmanrao, et al. [48], Srinivasarao et al. [49] Madhulatha, et al. [50] Have developed and analyzed inventory models with random replenishment (production).In their papers they assumed that the rate of deterioration is either constant are follows a weibull decay. However in some production systems that rate of deterioration may increase as along with time. For this type of items the lifetime may be approximated with a generalized Pareto distribution. Hence, in this paper we develop and analyze an EPQ model with a assumption that the production quantity is random and follows a Generalized Pareto distribution. It is also further assume the lifetime commodity as a random and follows a generalized Pareto distribution. The Generalized Pareto distribution includes exponential distribution and uniform distribution as limiting distributions. Using the differential equations the instantaneous state of inventory is derived under the assumptions that the shortages are fully backlogged. With suitable cost considerations the total cost of production is obtain and minimized for obtaining optimal values of production up and down time and production quantity. This model is extended to the case of without shortages.

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2. SENSITIVITY ANALYSIS OF THE MODEL IS PRESENTED ASSUMPTIONS OF THE MODEL

The following assumptions are made for developing the model.

i) The demand rate is a constant, which is $\lambda(t) = d$; t: time to deterioration (1)

ii) The production quantity is random and follows a Generalized pareto distribution having the probability density function

$f(t) = \frac{1}{\alpha}\left(1 - \frac{\beta t}{\alpha}\right)^{\frac{1}{\beta}-1}; (v \neq 0); 0 < t < \frac{\alpha}{\beta}$. Therefore, the instantaneous rate of replenishment is

$$k(t) = \frac{f(t)}{1-F(t)} = \frac{1}{\alpha - \beta t}; \alpha > 0, \beta > 0, 0 < t < \frac{\alpha}{\beta} \quad (2)$$

iii) Lead time is zero

iv) Cycle length, T is known and fixed

v) Shortages are allowed and fully backlogged

vi) A deteriorated unit is lost

vii) The lifetime of the item is random and follows a generalized Pareto distribution. Then the instantaneous rate of deterioration is

$$h(t) = \frac{1}{\lambda_1 - \lambda_2 t}; 0 < t < \frac{\lambda_1}{\lambda_2} \quad (3)$$

3. INVENTORY MODEL WITH SHORTAGES

Consider an inventory system in which the stock level is zero at time t=0. The Stock level increases during the period (0, t_1), due to excess of Production after fulfilling the demand and deterioration. The Production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2, the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3, the production starts again and fulfils the backlog after satisfying the demand. During (t_3, T), the back orders are fulfilled and inventory level reaches zero at the end of the cycle T. The Schematic diagram representing the instantaneous state of inventory is given in Figure 1.

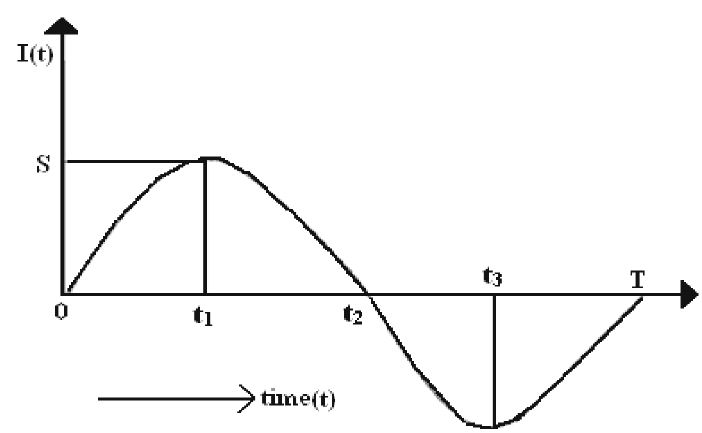

Fig 1: Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{d}{dt}I(t)+\frac{1}{(\lambda_1-\lambda_2 t)}I(t)=\frac{1}{\alpha-\beta t}-d; \quad 0\le t\le t_1 \quad (4)$$

$$\frac{d}{dt}I(t)+\frac{I(t)}{(\lambda_1-\lambda_2 t)}=-d\,; \quad t_1\le t\le t_2 \quad (5)$$

$$\frac{d}{dt}I(t)=-d\,; \quad t_2\le t\le t_3 \quad (6)$$

$$\frac{d}{dt}I(t)=\frac{1}{\alpha-\beta t}-d\,; \quad t_3\le t\le T \quad (7)$$

The solution of differential equations (4) to (7) using the initial conditions, I(0) = 0, I(t_1) = S,

I(t_2) = 0 and I(T) = 0, the on hand inventory at time 't' is obtained as

$$I(t)=S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-d\right)(\lambda_1-\lambda_2 u)^{\frac{-1}{\lambda_2}}du;\ 0\le t\le t_1 \quad (8)$$

$$I(t)=S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-d(\lambda_1-\lambda_2 t)\int_t^{t_1}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du;\ \ t_1\le t\le t_2 \quad (9)$$

$$I(t)=d[t_2-t];\ \ t_2\le t\le t_3 \quad (10)$$

$$I(t)=\frac{1}{\beta}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t}\right)+d(T-t);\ \ t_3\le t\le T \quad (11)$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t)=\int_0^t k(t)dt-\int_0^t \lambda(t)dt-I(t)\,,0\le t\le t_2$$

This implies

$$L(t)=\begin{cases}\ln\left(\frac{\alpha}{\alpha-\beta t}\right)^{\frac{1}{\beta}}-dt-S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du; \\ \quad 0\le t\le t_1 \\ \ln\left(\frac{\alpha}{\alpha-\beta t}\right)^{\frac{1}{\beta}}-dt-S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}}-(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_{t_1}^{t}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du; \\ \quad t_1\le t\le t_2\end{cases}$$

Stock loss due to deterioration in the cycle of length T is

$$L(T)=\ln\left(\frac{\alpha}{\alpha-\beta t1}\right)^{\frac{1}{\beta}}-dt$$

Ordering quantity Q in the cycle of length T is

$$Q=\int_0^{t_1}k(t)dt+\int_{t_3}^{T}k(t)dt=\frac{1}{\beta}\ln\left(\frac{\alpha(\alpha-\beta t_3)}{(\alpha-\beta t_1)(\alpha-\beta T)}\right) \quad (12)$$

$$S=(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\int_0^{t_1}\left(\frac{1}{\alpha-\beta u}-d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du \quad (13)$$

When t = t_3, then equations (10) and (11) become

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$I(t_3) = d(t_2 - t_3)$

and $I(t_3) = \frac{1}{\beta}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right) + d(T - t_3)$

Equating the equations and on simplification, one can get

$$t_2 = \frac{1}{d\beta} ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right) + T \tag{14}$$

Let $K(t_1, t_{2,} t_3)$ be the total cost per unit time. Since the total cost is thesum of the set-up cost, cost of the units, the inventory holding cost and shortage cost, the total Production cost per unit time becomes

$$K(t_1, t_{2,} t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt\right) \tag{15}$$

Substituting the values of I(t) and Q in equation (12), one can obtain $K(t_1, t_{2,} t_3)$ as

$$\begin{aligned} K(t_1, t_{2,} t_3) = {} & \frac{A}{T} + \frac{C}{T}\left(\frac{1}{\beta}\ln\left(\frac{\alpha(\alpha-\beta t_3)}{(\alpha-\beta t_1)(\alpha-\beta T)}\right)\right) + \frac{h}{T}\left\{\int_0^{t_1}\left[S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\right.\right. \\ & \left.\times \int_t^{t_1}\left(\frac{1}{\alpha-\beta u} - d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]dt + \int_{t_1}^{t_2}\left[S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - d(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\right. \\ & \left.\left.\times \int_{t_1}^{t}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}\, du\right]dt\right\} + \frac{1}{\beta}\int_{t_3}^{T}\ln\left(\frac{\alpha-\beta t}{\alpha-\beta T}\right)dt \\ & + \frac{\pi}{T}\left\{d\left[\int_{t_2}^{t_3}(t-t_2)dt\right] + \left[d + \frac{1}{\beta}\int_{t_3}^{T}\ln\left(\frac{\alpha-\beta t}{\alpha-\beta T}\right)dt\right]\right. \\ & \left. + \frac{1}{\beta}\left[\int_{t_3}^{T}\ln\left(\frac{\alpha-\beta t}{\alpha-\beta T}\right)dt\right]\right\} \end{aligned} \tag{16}$$

Substituting the values of 'S' and 't_2' from equations (13) and (14) in the total Production cost per unit time is (16) unit time is

$$\begin{aligned} K(t_1, t_3) = {} & \frac{A}{T} + \frac{C}{T}\left(\frac{1}{\beta}\ln\left(\frac{\alpha(\alpha-\beta t_3)}{(\alpha-\beta t_1)(\alpha-\beta T)}\right)\right) + \frac{h}{T}\left\{\int_0^{t_2}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_0^{t_1}\left(\frac{1}{\alpha-\beta u} - d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]dt\right. \\ & - \int_0^{t_1}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_t^{t_1}\left(\frac{1}{\alpha-\beta u} - d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]dt \\ & \left. - d\int_{t_1}^{t_2}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_{t_1}^{t}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]dt\right\} \\ & + \frac{\pi}{\beta T}\left\{\left[\left(T - \frac{\alpha}{\beta}\right) + \frac{1}{2d\beta}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right]\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right) + (t_3 - T)\right\} \end{aligned} \tag{17}$$

4 .OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL

In this section we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1 and t_3, we obtain the first order partial derivatives of $K(t_1, t_3)$ given in equation (17) with respect to t_1 and t_3 and equate them to zero. The condition for minimization of $K(t_1, t_3)$ is

$$D = \begin{vmatrix} \frac{\partial^2 K(t_1,t_3)}{\partial t_1^2} & \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} \\ \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} & \frac{\partial^2 K(t_1,t_3)}{\partial {t_3}^2} \end{vmatrix} > 0$$

Differentiating $K(t_1, t_3)$ given in equation (17) with respect to t_1 and equating to zero, one can obtain

$$\frac{C}{(\alpha-\beta t_1)} + h(\lambda_1 - \lambda_2 t_1)^{-\frac{1}{\lambda_2}}\left[\left[\left(\frac{1}{\alpha-\beta t_1} - d\right)\right]\left[\int_0^T (\lambda_1 - \lambda_2 t_1)^{\frac{1}{\lambda_2}} dt - \int_0^{t_1} (\lambda_1 - \lambda_2 t)^{\frac{1}{\lambda_2}} dt\right] + d\int_{t_1}^T (\lambda_1 - \lambda_2 t_1)^{\frac{1}{\lambda_2}} dt\right] = 0 \quad (18)$$

Differentiating $K(t_1, t_3)$ given in equation (17) with respect to t_3 and equating to zero, one can obtain

$$-\frac{C}{\alpha-\beta t_3} + h\big(\lambda_1 - \lambda_2 x(t_3)\big)^{\frac{1}{\lambda_2}}\left(\int_0^{t_1}\left(\frac{1}{\alpha-\beta u} - d\right)(\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du - d\int_{t_1}^T (\lambda_1 - \lambda_2 u)^{-\frac{1}{\lambda_2}} du\right) + \frac{\pi}{(\alpha-\beta t_3)}\left[T - t_3 + \frac{1}{d\beta}\ln\left(\frac{\alpha-\beta T}{\alpha-\beta t_3}\right)\right] = 0 \quad (19)$$

Solving equations (18) and (19) simultaneously, we obtain the optimal time at which production is to be stopped t_1^* of t_1 and the optimal time t_3^* of t_3 at which the production should be restarted after accumulation of backorders is obtained.

The optimum ordering quantity $Q^*$of Q in the cycle of length T is obtained by substituting the optimal values of t_1^*, t_3^* in equation (12).

5. NUMERICAL ILLUSTRATION

To expound the model developed, consider the case of deriving an optimal ordering quantity, production down time, production uptime and total cost for an edible oil manufacturing unit. Here, the product is deteriorating type and has random life time and assumed to follow a Generalized Pareto distribution. Based on the discussions held with the personnel connected with the production and marketing of the plant and the records, the values of different parameters are considered as A = Rs.1000/- C = Rs.10/- h =Re. 20/- π =Re. 8/-, =0.75, T = 12 months. For the assigned values of production parameters (α, β) = (50,3), deterioration rate parameters and (λ_1, λ_2) = (100,10). The values of parameters above are varied further to observe the trend in optimal policies, and the results obtained are shown in Table 1. Substituting these values, the optimal ordering quantity Q*, production uptime, production down time and total cost are computed and presented in Table 1.

Table 1

Optimal values of t^*_1, t^*_3, Q* and K* for different values of parameters

A	C	h	π	α	β	*d*	λ_1	λ_2	t_1	t_3	Q	K
1000	10	20	8	50	3	0.75	100	10	2.203	7.857	0.259	45.722
1005									2.199	7.847	0.259	46.137
1010									2.195	7.836	0.26	46.552
1015									2.191	7.826	0.26	46.967
	10.5								2.201	7.886	0.258	45.734
	11								2.2	7.915	0.257	45.746
	11.5								2.198	7.944	0.256	45.758
		20.2							2.203	7.867	0.259	45.347
		20.4							2.204	7.876	0.258	44.972
		20.6							2.205	7.886	0.258	44.597

			8.2						2.203	7.821	0.26	45.713
			8.4						2.203	7.784	0.262	45.705
			8.6						2.203	7.748	0.263	45.696
				50.2					2.204	7.864	0.256	45.72
				50.4					2.205	7.87	0.254	45.719
				50.6					2.207	7.877	0.251	45.717
					3.1				2.201	7.83	0.273	45.721
					3.2				2.199	7.801	0.288	45.721
					3.3				2.197	7.772	0.305	45.722
						0.76			2.207	7.868	0.259	45.215
						0.77			2.212	7.88	0.258	44.709
						0.78			2.216	7.891	0.258	44.202
							105		2.203	7.862	0.259	45.553
							110		2.203	7.865	0.259	45.414
							115		2.203	7.868	0.259	45.298
								10.2	2.203	7.856	0.259	45.762
								10.4	2.203	7.855	0.259	45.806
								10.6	2.202	7.854	0.259	45.853

From Table 1 it is observed that the deterioration parameters and production parameters have a tremendous influence on the optimal production times, ordering quantity and total cost.

When the ordering cost 'A' increases from 1000 to 1015, the optimal ordering quantity Q* increases from 0.259 to 0.26, the optimal production down time t_1 * decreases from 2.203 to 2.191, the optimal production uptime t_3 decreases from7.857 to 7.826, the total cost per unit time K*, increases from 45.722 to 46.967.

As the cost parameter 'C' increases from 10 to 11.5, the optimal ordering quantity Q* decreases from 0.259 to 0.256, the optimal production down time t_1 * decreases from 2.203 to 2.198, the optimal production uptime t_3 * increases from 7.857 to7.944, the total cost per unit time K*, increases from 45.722 to 45.758.

As the holding cost 'h' increases from 20 to 20.6, the optimal ordering quantity Q* decreases from 0.259 to 0.258, the optimal production down time t_1 * increases from 2.203 to 2.205, the optimal production uptime t_3 * increases from 7.857 to 7.886, the total cost per unit time K*, decreases from 45.722 to 44.597.

As the shortage cost 'π' increases from 8 to 8.6, the optimal ordering quantity Q* * increases from 0.259 to 0.263, the optimal production down timet$_1$ * increases from 2.203 to 2.203, the optimal production uptime t_3 * decreases from 7.857 to 7.748. The total cost per unit time K*, decreases from 45.722 to 45.696.

As the production parameter 'α' varies from 50 to 50.6, the optimal ordering quantity Q* decreases from 0.259 to 0.251, the optimal production down time t_1 * increases from 2.203 to 2.207, the optimal production uptime t_3 * increases from 7.857 to 7.877, the total cost per unit time K*, decreases from 45.722 to 45.717.

Another production parameter 'β' varies from 3 to 3.3, the optimal ordering quantity Q* increases from 0.259 to 0.305, the optimal production down time t_1 * decreases from 2.203 to 2.197, the optimal production uptime t_3 * decreases from 7.857 to 7.772, the total cost per unit time K*, increases from 45.721 to 45.722.

As the deterioration rate parameter 'λ_1'varies from 100 to 115, the optimal ordering quantity Q* increases from 0.259 to 0.259, the optimal production down time t_1 * increases from 2.203 to 2.203, the optimal production uptime t_3 * increases from 7.857 to7.868, the total cost per unit time K*, decreases from 45.722 to 45.298.

Another deterioration rate parameter 'λ_2' varies from 5 to 6.5, the optimal ordering quantity Q* increases from 0.259 to 0.259, the optimal production down time t_1 * decreases from 2.203 to 2.202, the optimal production uptime t_3 * decreases from 7.857 to 7.854, the total cost per unit time K*, increases from 45.722 to 45.853.

As the demand rate 'd' increases from 0.75 to 0.78, the optimal ordering quantity Q* decreases from 0.259 to 0.258, the optimal production down time t_1 * increases from 2.203 to 2.216, the optimal production uptime t_3 * increases from 7.857 to 7.891, the total cost per unit time K*, decreases from 45.722 to 44.202.

6. SENSITIVITY ANALYSIS OF THE MODEL

To study the effects of changes in the parameters on the optimal values of production down time, production uptime, optimal ordering quantity and total cost, sensitivity analysis is performed taking the values of the parameters as A = Rs.1000/- C = Rs.10/- h =Re. 20/-, π=Re. 8/-, d =0.75, T = 12 months. For the assigned values of production parameters (α,β)=(50,3), deterioration parameters (λ_1, λ_2) = (100,10). Sensitivity analysis is performed by changing the parameter values by -6%, - 4%, -2%, 0%, 2%,4%,6%. First changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values of production down time, production uptime, optimal ordering quantity and total cost are computed. The results are presented in Table2 and Fig 2. The relationships between parameters, costs and the optimal values are shown in Fig.2. From Table 2, it is observed that the deteriorating parameters (λ_1, λ_2) have less effect on production down time t_1 *, production up time t_3 * and significant effect on optimal ordering quantity and total cost. Decrease in unit cost C results decrease in production down time t_1 *and increase in production up time t_3 *,decrease in optimal ordering quantity Q* and increase in total cost K*. The increase in production rate parameters. Increase in holding cost h results significant variation in optimal ordering quantity Q* and decrease in total cost K*. The increase in shortage cost results less effect on optimal ordering quantity Q* and total cost K*.

Table 2

Sensitivity analysis of the model – without shortages

Parameters/Costs	Optimal policies	Change in parameters						
		-6%	-4%	-2%	0%	2%	4%	6%
A	t_1^*	2.247	2.233	2.218	2.203	2.188	2.173	2.158
	t_3^*	7.983	7.941	7.899	7.857	7.816	7.774	7.733
	Q^*	0.255	0.257	0.258	0.259	0.26	0.261	0.263
	K^*	40.74	42.401	44.061	45.722	47.382	49.043	50.703
C	t_1^*	2.204	2.204	2.203	2.203	2.202	2.202	2.201

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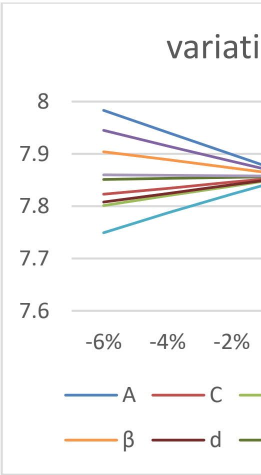
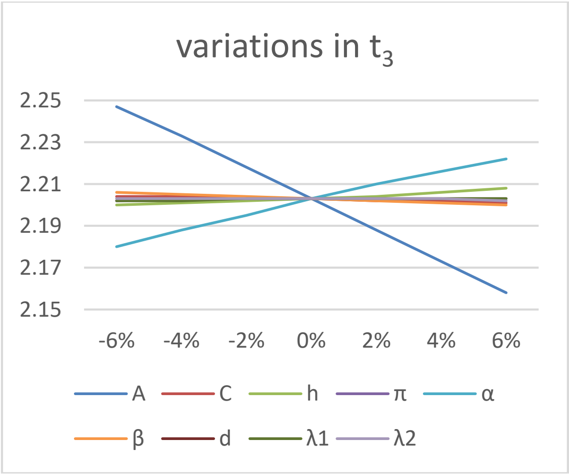
	t_3^*	7.823	7.834	7.846	7.857	7.869	7.88	7.892
	Q^*	0.26	0.26	0.259	0.259	0.259	0.258	0.258
	K^*	45.707	45.712	45.717	45.722	45.727	45.732	45.737
h	t_1^*	2.2	2.201	2.202	2.203	2.204	2.206	2.208
	t_3^*	7.801	7.82	7.838	7.857	7.876	7.895	7.914
	Q^*	0.261	0.26	0.26	0.259	0.258	0.258	0.257
	K^*	47.973	47.222	46.472	45.722	44.972	44.222	43.471
π	t_1^*	2.202	2.203	2.203	2.203	2.203	2.203	2.203
	t_3^*	7.945	7.915	7.886	7.857	7.828	7.799	7.77
	Q^*	0.256	0.257	0.258	0.259	0.26	0.261	0.262
	K^*	45.741	45.735	45.728	45.722	45.715	45.708	45.701
α	t_1^*	2.18	2.188	2.195	2.203	2.21	2.216	2.222
	t_3^*	7.749	7.787	7.823	7.857	7.889	7.919	7.947
	Q^*	0.307	0.289	0.273	0.259	0.246	0.235	0.225
	K^*	45.745	45.737	45.729	45.722	45.715	45.708	45.701
β	t_1^*	2.206	2.205	2.204	2.203	2.202	2.201	2.2
	t_3^*	7.904	7.889	7.873	7.857	7.841	7.824	7.807
	Q^*	0.238	0.245	0.252	0.259	0.267	0.275	0.285
	K^*	45.723	45.723	45.722	45.722	45.721	45.721	45.721
d	t_1^*	2.182	2.189	2.196	2.203	2.209	2.216	2.223
	t_3^*	7.808	7.824	7.841	7.857	7.874	7.891	7.909
	Q^*	0.26	0.26	0.26	0.259	0.259	0.258	0.258
	K^*	48.001	47.241	46.481	45.722	44.962	44.202	43.443
λ_1	t_1^*	2.202	2.202	2.203	2.203	2.203	2.203	2.203
	t_3^*	7.851	7.853	7.855	7.857	7.859	7.861	7.862
	Q^*	0.259	0.259	0.259	0.259	0.259	0.259	0.259
	K^*	45.985	45.888	45.801	45.722	45.65	45.584	45.523
λ_2	t_1^*	2.203	2.203	2.203	2.203	2.203	2.203	2.202
	t_3^*	7.86	7.859	7.858	7.857	7.856	7.855	7.854
	Q^*	0.259	0.259	0.259	0.259	0.259	0.259	0.259
	K^*	45.614	45.648	45.684	45.722	45.762	45.806	45.853





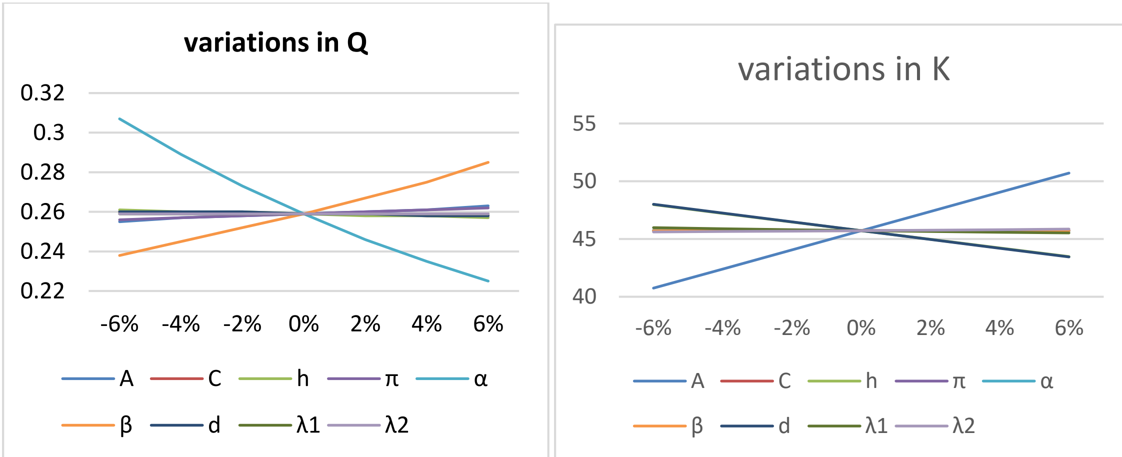


Fig 2: Relationship between optimal values and parameters with shortages

7.INVENTORY MODEL WITHOUT SHORTAGES

In this section, the inventory model for deteriorating items without shortages is developed and analysed. Here, it is assumed that shortages are not allowed and the stock level is zero at time t = 0. The stock level increases during the period (0, t_1), due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1,T). At time T, the inventory reaches zero. The Schematic diagram representing the instantaneous state of inventory is given in Figure 3.

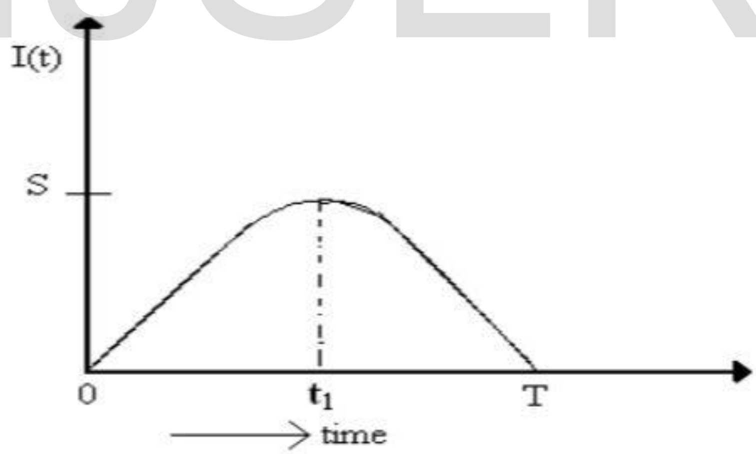


Fig 3: Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{d}{dt}I(t) + h(t)I(t) = \frac{1}{\alpha-\beta u} - d; \qquad 0 \le t \le t_1 \qquad (20)$$

$$\frac{d}{dt}I(t) + \frac{I(t)}{h(t)} = -d; \qquad t_1 \le t \le t_2 \qquad (21)$$

where, h(t) is as given in equation (3), with the initial conditions, I(0) = 0, I(t1) = S, and I(T) = 0. Substituting h (t) given in equation (3) in equation (20) and (21) and solving the differential equations, the on-hand inventory at time 't' is obtained as

$$I(t) = S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}} \int_t^{t_1} \left[\frac{1}{\alpha-\beta u} - d\right](\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du; \quad 0 \le t \le t_1 \tag{22}$$

$$I(t) = S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - d(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}} \int_{t_1}^{t} (\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du \quad t_1 \le t \le T \tag{23}$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t k(t)dt - \int_0^t \lambda(t)dt - I(t) \qquad 0 \le t \le T$$

This implies

$$L(t) = \begin{cases} \ln\left(\frac{\alpha}{\alpha-\beta t}\right)^{\frac{1}{\beta}} - dt - S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - (\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}} \int_0^{t_1} \left(\frac{1}{\alpha-\beta u} - d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du; 0 \le t \le t_1 \\ \ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}} - dt - S\left(\frac{\lambda_1-\lambda_2 t}{\lambda_1-\lambda_2 t_1}\right)^{\frac{1}{\lambda_2}} - d(\lambda_1-\lambda_2 t) \int_{t_1}^{t} (\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du; t_1 \le t \le T \end{cases}$$

Ordering quantity Q in the cycle of length T is

$$Q = \int_0^{t_1} k(t)dt = \ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}} \tag{24}$$

From equation (22) and using the condition I (0) = 0, we obtain the value of 'S' as

$$S = (\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}} \int_0^{t_1} \left[\frac{1}{\alpha-\beta u} - d\right](\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}} du \tag{25}$$

Let $K(t_1)$ be the total cost per unit time. Since the total cost is the sum of the set-up cost, cost of the units, the inventory holding cost. Therefore, the total cost is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right) \tag{26}$$

Substituting the value of I (t), Q and S given in equation's (22), (23), (24) and (25) in equation (26) and on simplification, we obtain $K(t_1)$ as

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\left(\ln\left(\frac{\alpha}{\alpha-\beta t_1}\right)^{\frac{1}{\beta}}\right)$$

$$+\frac{h}{T}\Bigg\{\int_0^{t_1}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_0^{t_1}\left(\frac{1}{\alpha-\beta u}-d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]\right]dt$$

$$-\int_0^{t_1}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_t^{t_1}\left(\frac{1}{\alpha-\beta u}-d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]\right]dt$$

$$+\int_{t_1}^{T}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_0^{t_1}\left(\frac{1}{\alpha-\beta u}-d\right)(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]\right]dt$$

$$-\int_{t_1}^{T}\left[(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}\left[\int_{t_1}^{t}(\lambda_1-\lambda_2 u)^{-\frac{1}{\lambda_2}}du\right]\right]dt\Bigg\} \quad (27)$$

8.OPTIMAL PRICING AND ORDERING POLICIES OF THE MODEL WITHOUT SHORTAGES

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, we equate the first order partial derivatives of $K(t_1)$ with respect to t_1 equate them to zero. The condition for minimum of $K(t_1)$ is $\frac{d^2K(t_1)}{dt_1^2}>0$

Differentiating $K(t_1)$ with respect to t1 and equating to zero we get

$$\frac{C}{(\alpha-\beta t_1)}+h(\lambda_1-\lambda_2 t_1)^{-\frac{1}{\lambda_2}}\left[\left[\left(\frac{1}{\alpha-\beta t_1}-d\right)\right]\left[\int_0^{T}(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}dt-\int_0^{t_1}(\lambda_1-\lambda_2 t)^{\frac{1}{\lambda_2}}dt\right]+d\int_{t_1}^{T}(\lambda_1-\lambda_2 t_1)^{\frac{1}{\lambda_2}}dt\right]=0 \quad (28)$$

Solving the equation (28), we obtain the optimal time at which the production is to be stopped at t_1^* of t_1. The optimal ordering quantity Q* of Q in the cycle of length T is obtained by substituting the optimal value of t_1 in equation (24).

9. NUMERICAL ILLUSTRATION OF MODEL WITHOUT SHORTAGES

Consider, the product is deteriorating type and has random life time and assumed to follow a Generalized Pareto distribution. The values of different parameters are considered as A = Rs.1000/- C = Rs.100/- h =Re. 10/-, d = 0.75, T = 12 months. For the assigned values of production parameters (α, β) = (52.5,54), deterioration parameters (λ_1, λ_2) = (200,15). The values of above parameters are varied further to observe the trend in optimal policies and the results obtained are shown in Table 3. Substituting these values, the optimal ordering quantity Q*, production uptime, production down time and total cost are computed and presented in Table 3.

From Table 3 it is observed that the deterioration rate parameters and production rate parameters have a tremendous influence on the optimal production time, optimal ordering quantity and total cost.

Table 3

Optimal values of t1 *, Q* and K* for different values of parameters

A	C	h	α	β	d	λ_1	λ_2	t_1	Q	K
1000	100	10	50	2	0.75	200	15	3.486	0.388	44.066

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1010								3.474	0.387	44.892
1020								3.462	0.386	45.717
1030								3.45	0.385	46.543
	101							3.484	0.387	44.097
	102							3.481	0.387	44.127
	103							3.478	0.387	44.158
		10.5						3.514	0.389	41.96
		11						3.542	0.391	39.854
		11.5						3.571	0.393	37.749
			50.5					3.488	0.386	44.044
			51					3.491	0.384	44.022
			51.5					3.493	0.382	44.001
				2.1				3.477	0.415	44.298
				2.2				3.469	0.442	44.519
				2.3				3.462	0.467	44.731
					0.76			3.494	0.388	43.497
					0.77			3.502	0.389	42.928
					0.79			3.511	0.389	42.358
						205		3.488	0.388	43.953
						210		3.489	0.388	43.855
						215		3.49	0.388	43.77
							15.5	3.485	0.387	44.183
							16	3.483	0.387	44.335
							16.5	3.48	0.387	44.565

10. SENSITIVITY ANALYSIS OF THE MODEL WITHOUT SHORTAGES

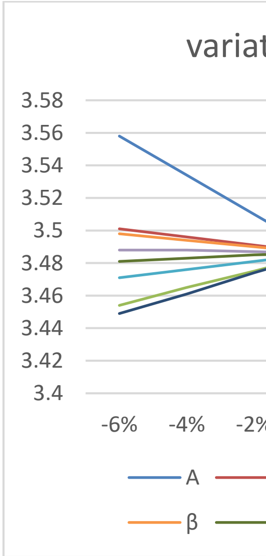
To study the effects of changes in the parameters on the optimal values of production time, optimal ordering quantity and total cost, sensitivity analysis is performed taking the values of the parameters as A = Rs.1000/- C = Rs.100/- h =Re. 10/-,d=0.75, T = 12 months. For the assigned values of production rate parameters (α, β) = (50,2), deterioration rate parameters (λ_1, λ_2) = (200,15). Sensitivity analysis is performed by changing the parameter values by –6%, -4%, -2%,0%,2%,4% and 6%. First, changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values of production time, optimal ordering quantity and total cost are computed. The results are presented in Table 4. The relationships between parameters, costs and the optimal values are shown in Fig.3.

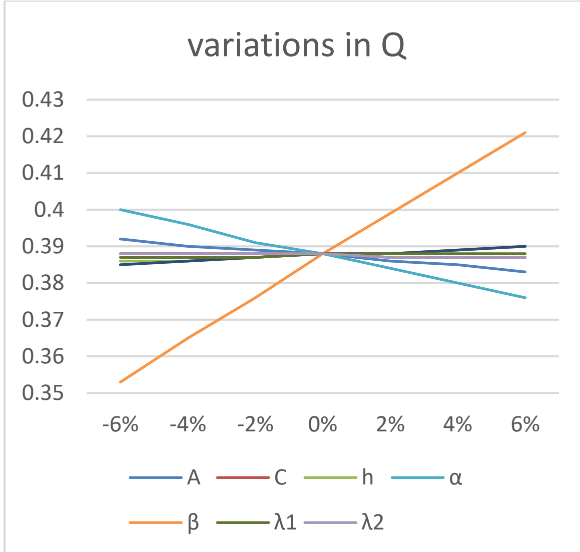
From Table 4, it is observed that variation in the deterioration parameters (λ_1 ,λ_2) has considerable effect on production time t_1* , optimal ordering quantity Q* and total cost K*.Similarly , variation in deterioration parameters (λ_1 ,λ_2) has slight effect on production time t_1* , optimal ordering quantity Q* and significant effect on total cost K*.The decrease in unit cost 'C' results in decrease in production time t_1 * , and decrease in optimal ordering quantity Q*and increase in total cost K*. The increase in holding cost h has significant effect on increase in optimal values of production time t_1 *, optimal ordering quantity Q* and decrease in total cost K*.

Table 4

Sensitivity analysis of the model – without shortages

Parameters/Costs	Optimal Values	Change in parameters						
		-6%	-4%	-2%	0%	2%	4%	6%
A	t_1^*	3.558	3.534	3.51	3.486	3.462	3.438	3.414
	Q^*	0.392	0.39	0.389	0.388	0.386	0.385	0.383
	K^*	39.113	40.764	42.415	44.066	45.717	47.368	49.019
C	t_1^*	3.501	3.496	3.491	3.486	3.481	3.476	3.471
	Q^*	0.388	0.388	0.388	0.388	0.387	0.387	0.387
	K^*	43.882	43.943	44.005	44.066	44.127	44.189	44.249
h	t_1^*	3.454	3.465	3.475	3.486	3.497	3.508	3.519
	Q^*	0.386	0.386	0.387	0.388	0.388	0.389	0.39
	K^*	46.595	45.752	44.909	44.066	43.223	42.381	41.538
α	t_1^*	3.471	3.476	3.481	3.486	3.491	3.495	3.499
	Q^*	0.4	0.396	0.391	0.388	0.384	0.38	0.376
	K^*	44.208	44.159	44.112	44.066	44.022	43.979	43.938
β	t_1^*	3.498	3.494	3.49	3.486	3.482	3.479	3.476
	Q^*	0.353	0.365	0.376	0.388	0.399	0.41	0.421
	K^*	43.776	43.874	43.971	44.066	44.16	44.252	44.343
d	t_1^*	3.449	3.461	3.474	3.486	3.498	3.511	3.523
	Q^*	0.385	0.386	0.387	0.388	0.388	0.389	0.39
	K^*	46.628	45.774	44.92	44.066	43.212	42.358	41.504
λ_1	t_1^*	3.481	3.483	3.485	3.486	3.487	3.488	3.489
	Q^*	0.387	0.387	0.387	0.388	0.388	0.388	0.388
	K^*	44.448	44.297	44.172	44.066	43.974	43.893	43.82
λ_2	t_1^*	3.488	3.488	3.487	3.486	3.485	3.484	3.483
	Q^*	0.388	0.388	0.388	0.388	0.387	0.387	0.387
	K^*	43.905	43.954	44.007	44.066	44.133	44.21	44.3





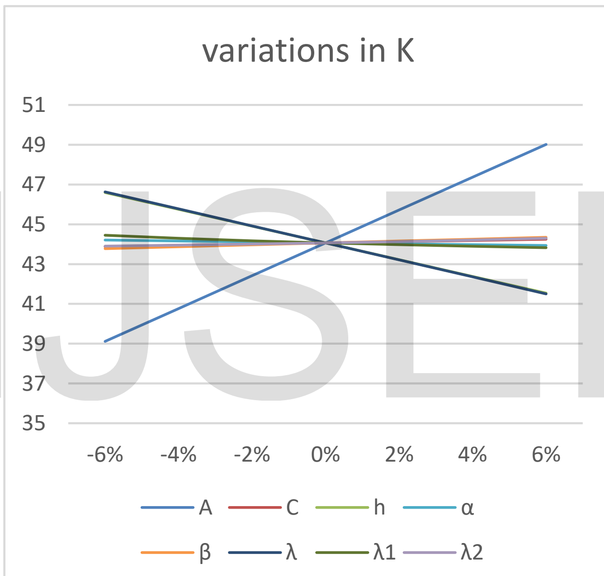


Fig 4: Relationship between optimal values and parameters without shortages

11. CONCLUSIONS

This paper introduces a new EPQ model with random production having Generalized Pareto production rate and Generalized Pareto rate of decay, having constant demand. Here the production process as well as lifetime commodity are characterizing through Generalized Pareto processes in order to match their statistical characteristic suitable with the realistic situation. Generalized Pareto distribution can include exponential distribution as limiting case and uniform distribution as a particular case. Further the Generalized Pareto rate of production and decay can characterise the increasing rates of the production as well deterioration. In many production process the production rate increases with time similarly for deteriorating items such as seafood's, oils chemicals, the rate of deterioration increases with time. The instantaneous state of inventory under the assumption that shortages are allowed and fully backlogged is derived. The optimal production up and down times and optimal production quantity are derived. The sensitivity analysis of the model with respect to the changes in parameters and cost has revealed that the random production and deterioration have significant influence on optimal values of the production schedule and production quantity. This model also includes some earlier models as particular cases. In this model we assumed that the money values is constant thought the cycle time, it is also possible to extended this model with inflations and multi commodity items which will be taken up elsewhere.

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